On the analysis and optimal asset allocation of pension funds in regret theoretic framework

1. Introduction

The major contribution of this paper lies in the use of regret theory to analyse the optimal asset allocation of a pension fund that maximises the expected *modified* utility of its final wealth. Unlike the standard expected utility framework in which a pension fund manager independently considers only the investment choice that he makes and performs utility maximisation without any recourse to other investment choices that could have been made, regret theory gives room for the fund manager to account for his investment choice as well as other feasible investment choices that could have made. In essence, the fund manager experiences regret if the outcome of his investment choice is worse than the outcome of at least one of his forgone alternatives, and he rejoices if otherwise. Because of the anticipation of future regret, we set up the objective function—the expected modified utility of the fund's final wealth— in such a way as to incorporate regret function. The presence of this regret function distinguishes regret theory from the traditional expected utility framework. In this light, therefore, we develop a set-up aimed at examining the extent to which the anticipation of future regret influences the choice and optimal asset allocation of a pension fund.

Accessible literature on asset allocation problems and optimal financial portfolios for pension funds almost completely neglects regret theory and widely favors expected utility maximisation. In addition to other limitations and violations of the traditional expected utility theory, so elegantly demonstrated and documented in the behavioral economics literature, our major discontent with the theory is that it assumes individuals consider each possible outcome independently of other outcomes. This can be interpreted to mean that fund investment managers care only about their investment choices. However, as has been demonstrated in the behavioral finance literature, fund managers do experience regret whenever alternative investment choices yield better returns ex-post. Even though they ex-ante felt convinced that their investment decisions were optimal, fund managers still harbor a feeling of regret for not having made the right investment decisions whenever their ex-post returns on investments turn out to be worse off. As a simple illustration, consider a fund manager who can receive a \$5 return on investment for each dollar invested in the debt capital market and either a \$7.5 or \$3.5 return on investment for each dollar invested in the equity capital market. If he takes a huge position in the equity capital market and finally receives a \$3.5 for each dollar invested in equity, he may almost surely experience a feeling of regret for getting less than he would have gotten if he had taken little or no position in the equity capital market. Next time, this experience will shape his investment decisions and therefore make him averse to regret and this aversion will in turn force him to incorporate regret into his decision making process. This idea is well-documented in the so called behavioral decision theory under uncertainty. Unfortunately, however, nowhere has it been used to analyse the optimal asset allocation of a pension fund.

The concept of regret theory is intuitively straightforward. Regret is a cognitively mediated emotion of pain and anger when people observe that they took a bad decision ex-ante and could have taken an alternative decision with better outcome. In capital markets, people experience regret when their investments give a worse performance than an alternative investment they could have easily chosen. This, for instance, is in contrast with disappointment, which is experienced when a negative outcome happens relative to prior expectation. Regret is strongly associated with a feeling of responsibility for the choice that has been made and is known to influence decision-making under uncertainty. Regret is a powerful negative emotion; the anticipation of future regret is so strong that it forces even Harry Markowitz to turn against his very own Nobel winning asset allocation theory when confronted with a financial decision on his pension plan. His quote: 'I should have computed the historical covariance of the asset classes and drawn an efficient frontier. Instead I visualized my grief if the stock market went way up and I wasn't in it—or if it went way down and I was completely in it. My intention was to minimize future regret, so I split my pension scheme contributions 50-50 between bonds and equities.' 'Harry Markowitz. As quoted in Zweig, 1998, 'America's top pension fund', Money, 27, page 114' [3]. This gives support and adds credibility to the claim that regret does influence optimal investment decision of a pension fund. Anticipation of future experience of regret may lead individuals to make decisions that contrast the expected utility paradigm. This assertion will be investigated in the context of the optimal asset allocation of a pension fund in the course of this research.

Regret theory, due to Bell (1982) and Loomes and Sugden (1982), proposes a normative theory of choice under uncertainty that explains many observed violations of the axioms that the traditional expected utility theory is built upon. Regret theory involves the regret or rejoice that a person can feel when he gets outcome x instead of outcome y. The theory assumes that people are rational but base their decisions not only on expected payoffs or utility but also on expected regret, so that they try to anticipate future regret and consistently incorporate it into their investment decisions. The incorporation of regret yields a modified utility and people reach their investment decisions by maximizing the expected value of this modified utility. This makes the theory suitable for analysing the optimal asset allocation of a pension fund.

Unlike other institutional investors, the case of pension funds requires the introduction of two new characteristics: (i) the different behaviors of the fund wealth during the accumulation Ac and decumulation Dc phases, and (ii) the mortality risk. Also, regret risk is required because we are working in a regret theoretic framework. So, this thesis considers three dimensions of risk: traditional risk (volatility of final wealth), regret risk and mortality risk. To the best of our knowledge, no work on optimal asset allocation has ever simultaneously taken these risks into account to study the optimal asset allocation of a pension fund. The only work, at least to our knowledge, which considers these risks in pension fund research and asset allocation theory, does not consider them all at once. For instance, Bajeux-Besnainou and Jordon [5] consider only volatility risk, Michenaud and Solnik [3] consider volatility risk and regret risk and Battocchio, Menoncin and Scaillet [4] consider volatility risk and mortality risk.

Michenaud and Solnik [3] study the currency hedging techniques for foreign assets in a regret theoretic framework and derive some interesting implications for long and short hedging positions when a foreign currency appreciates or depreciates ex-post. In contrast, our methodology allows the derivation of approximate closed-form solutions for the optimal investment choices available to a pension fund. While the intuition of applying regret theory to asset allocation is not new, this is the first time/one of the few times that a formal regret theoretic approach is applied to a pension fund with mortality risk.

As we motivated above, regret is a major factor when making investment choices because institutional investors, more often than not, care about their choices relative to other strategies they could have employed. Although there has been observed evidence in favor of the influence of regret on decision-making under uncertainty as well as the axiomatic and normative appeal of regret theory for investment choices, it is surprising that the theory has caught only little attention in the field of finance, Michenaud and Solnik [3]. For instance, Braun, Mitchell and Volkman [6] apply regret theory to asset allocation in defined contribution pension schemes. They find that an investor who takes regret into account will hold more risky assets (stocks) when the equity premium is low but less risky assets when the equity premium is high. Mitchell and Muermann [7] apply regret theory to demand for insurance. Dodonova and Khoroshilov [8] apply a pseudo regret theory to asset pricing. Michenaud and Solnik [9] apply regret theory to portfolio optimisation. All these models offer comparative statics or approximate explicit solutions for investment rules outside the case of a pension fund.

In this paper, instead, we provide approximate explicit optimal solutions for investment rules within the context of regret theory in the case of a pension fund which manages employees' contributions towards retirement. In particular, during the active years of the employees, the fund wealth increases because of the contributions that the employees make towards retirement while, after retirement, the fund wealth decreases because of the pension payments that the pension fund makes to the retired employees. Following Battocchio, Menoncin and Scaillet [4], we suppose that a representative employee has no other choice at the retirement date than to receive a pension until the death time τ, which we assume to be stochastic. The pension fund then maximises the expected modified utility of its final wealth, in anticipation of future regret.

In our model the contribution and pension rates are constant and linked by a feasibility condition that guarantees the convenience of both the pension fund and the representative employee to amicably enter the pension contract. We argue why this feasibility condition must hold and derive its approximate closed-form expression under the assumption that the death time τ follows a log-logit distribution. We emphasise that our result is quite different from the closed form expression obtained under the assumption of a Gompertz-Makeham and Weibull distributed death time τ in Battocchio, Menoncin and Scaillet [4], and remark that our motivation for this choice of distribution for the death time τ stems from the fact that death-survival analyses for a random death time are best done under the assumption of a log-logit distribution [10].

To summarise, in addition to other important results, our major contribution in this paper is systematic. We integrate regret into a well-defined objective function and this allows us to derive optimal investment strategies that reflect the risk and regret aversion of a pension fund.

To this end, the paper flows as follows. Chapter 2 presents the financial model for the pension fund and explains some very important concepts that will aid the understanding of other ideas presented in subsequent chapters. Chapter 3 discusses the importance **of regret theory** for pension fund decisions and describes our modeling framework as well as the computation of the feasibility condition on the contribution and pension rates when the death time follows a Log-logit distribution. Chapter 4 presents the objective function for the pension fund and the computation of the optimal allocation rule. Chapter 5 discusses the main practical implications of our results for the effective management of a pension fund and concludes with direction to future research.

2. Essential Concepts

Here we present the notion of financial markets in relation to the pension fund industry and describe some relevant concepts that will make clear the arguments presented in the subsequent chapters of this research.

2.1 Financial Markets

Financial markets serve as a transaction point where investors trade securities, commodities and other transposable financial instruments such as currencies and derivatives at prices that reflect demand and supply. Investors can be private or institutional investors while financial markets can be capital, money, derivatives, commodities, foreign exchange and insurance markets.

2.1.1 Private and Institutional Investors

Private investors are individuals who participate in transaction activities in the financial markets with limited initial capital while institutional investors comprise large corporations that engage in buy-side deals with large initial capital and restricted protective regulations. Institutional investors account for a majority of overall volume of trades in the financial markets and are known to include pension funds, hedge funds, insurance companies, brokerages, mutual funds, investment banks and asset management firms etc. We dwell mainly on the investment strategy of pension funds institutional investors because that is the basis of this research.

2.1.2 Pension Funds

Pension funds are important institutional investors that provide retirement income and benefits to their clients/subscribers. Their presence is especially felt in the money and capital markets where sell-side investors such as private and public enterprises as well as governments come to raise short- and long-term funds to finance business operations and capital expenditures. Pension funds worldwide hold over $20 trillion in assets and therefore dominate other institutional investors in terms of investments in assets [11]. This suggests the importance of pension funds and hence the need for them to be studied.

Researchers have studied different types of pension plans, some of which are:

- Defined-benefit plan
- Defined-contribution plan

Defined-Benefit and Defined-Contribution Plans

While defined-benefit pension plans are employer-sponsored plans in which a retired employee receives specific retirement benefits based on years of service and salary history, defined- contribution pension plans allow the employee to make seasonal contributions to the fund but there is no way the employee can know the specific retirement benefits because everything depends on the rate of return of the invested funds. Another important distinction between both pension plans is that the employee sets up an account and makes the investment decisions in the case of defined-contribution pension plans while the employer makes all investment decisions in the case of defined-benefit pension plans. Thus, the employee bears the investment risk in defined-contribution pension plans while the employer manages the investment portfolio and bears the investment risk in the case of defined-benefit plans. Annuities are defined-benefit plans that have fixed monthly payments at the age of retirement while 401k plans are defined-contribution plans that allow tax-deferred income to finance retirement benefits.

Unlike other institutional investors, the analysis of pension funds requires the introduction of three unique characteristics:

- The behavior of the fund wealth in the accumulation phase (Ac)
- The variation of the fund wealth in the decumulation phase (Dc)
- The mortality risk of the subscriber

Accumulation and Decumulation Phases

The representative subscriber makes contributions to the pension fund during the accumulation phase and so the fund wealth swells while the pension fund makes mandatory payments (pensions) to the subscriber in the decumulation phase and so the fund wealth shrinks.

Researchers have established the link between contributions, in the accumulation phase, and pensions, in the decumulation phase, inside either the defined-benefit plan or the defined contribution plan. In a defined benefit plan, the employer fixes benefits in advance, and contributions are designed to maintain the fund in balance. In a defined contribution plan, contributions are fixed but benefits depend on the returns of the invested funds. The model presented here concerns the case of a pension fund that offers its subscribers a deterministic pension plan. The deterministic pension plan is such that the subscribers make a constant contribution to the fund while the pension fund pays a constant pension to the subscribers. This is the so called 'Cash Balance Plan', which is especially prominent in the US.

Mortality Risk

In any pension plan, mortality risk is the risk that an active subscriber who is accumulating his pension benefits will die earlier than expected. This contrasts longevity risk, which is the risk that an inactive member with pensions in payment will live longer than expected. Essentially, mortality risk is restricted to accumulation phase while longevity risk is restricted to decumulation phase. As we can quickly infer therefore, longevity risk is worse than mortality risk because it ultimately leads to the depletion of the wealth of the pension fund if it carries on for a very long time.

2.2 The Financial Market Model for a Pension Fund

We follow Brennan, Schwartz and Lagnado [12] and Menoncin and Scaillet [3] and consider a financial market with one risky-asset class (common stock) and one riskless-asset class (T-bill) whose rates of return are $\frac{dS}{S}$ and r respectively, where r is the short term interest rate which predicts the expected return on common stock [13]. If we assume also that the dividend yield δ on common stocks influences returns on the risky-asset class, then the joint price process follows

$$\frac{dS}{S} = \mu_S dt + \sigma_S dz_S, \quad S(t_0) = S_0$$

$$\frac{dG}{G} = rdt, \quad G(t_0) = G_0$$

$$\begin{cases} dr = \mu_r dt + \sigma_r dz_r \\ d\delta = \mu_\delta dt + \sigma_\delta dz_\delta \end{cases}$$

where the parameters $\mu_i, \ \sigma_i \ (i = r, \delta, S)$ are at most functions of the variables r, δ, S, and dz_i are increments due to Weiner process. S_0 and G_0 are deterministic positive variables that represent the initial prices of the risky and riskless asset classes while S and G are their prices at time $t > 0$.

2.2.1 Contributions and Pensions

Contributions and pensions occur in two phases. The subscriber makes contributions to the fund in the accumulation phase (before retirement) while the pension fund pays pensions to the subscriber in the decumulation phase (after retirement and till death). We assume that the contributions and pensions are constant, and the retirement date T is set by employment law so that it does not vary with the employee preference.

If $U(t)$ denotes the total amount of contributions to the fund and $V(t)$ denotes the total amount of pensions paid by the fund, then $U(t)$ and $V(t)$ follow the ordinary linear differential equations

$$dU(t) = udt$$

$$dV(t) = vdt$$

where $u > 0$ and $v > 0$ are constant and do not vary with time. The pensions are paid until the death time of the subscriber and do not depend on the investment performance of the fund. As we remarked, our framework combines characteristics of both defined-benefit and defined-contribution schemes. Eastern European countries as well as Switzerland are the most popular proponents of our framework.

2.2.2 The Feasibility Condition

This is the condition that has to be satisfied before the subscriber and the pension fund can sign the pension contract in the first place. For this reason, the pension fund cannot freely dictate the contributions and pensions while the subscriber cannot solely dictate the pensions. The contributions and pensions cannot be chosen separately. The subscriber and the pension fund have to agree on the contributions and pensions simultaneously.

When the subscriber enters the fund, he anticipates that the expected present value of all pensions cannot be lower than the expected present value of all contributions. Similarly, the pension fund signs on the subscriber when it is convinced that the expected present value of all pensions cannot be more than the expected present value of all contributions.

Money enters or leaves the fund according to the rate

$$m(t) = \frac{dU(t)}{dt}\mathbb{I}_{t<T} - \frac{dV(t)}{dt}\mathbb{I}_{t\geq T}$$

or

$$m(t) = u\mathbb{I}_{t<T} - v(1 - \mathbb{I}_{t<T}),$$

where

$$\mathbb{I}_{t<T} = \begin{cases} 1, & \text{if } t < T \\ 0, & \text{if } t \geq T \end{cases}$$