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Profile	I am currently working as a Senior Consultant in the capital markets division of a big consulting firm. I possess the following academic qualifications - • MSc Quantitative Finance and Financial Engineering, Manchester Business School • Post Grad Diploma in Industrial Management (MBA) • Bachelors of Engineering (Electrical I had my project internship with ABN Amro Bank Amsterdam. I have interest in Business, management, strategy, statistics, economics, financial analysis, derivatives, capital markets, trading, econometrics, stochastic calculus, bond mathematics and interest rate derivatives. In my free time I generally play with my 2 year old daughter and spend time with the family. I am a big movie buff, I enjoy reading and playing computer games as well.
Subjects	Business, Management, Mathematics, Econometrics, Finance, Statistics

Title	A study of Interest Rate Models
Description	The purpose of this study is to perform an empirical analysis of the various interest rate models. However I have confined myself to the boundaries of only the short rate models.
Keywords	Finance, interest rates, free finance essay, free finance paper, free finance assignment, free finance coursework, free finance analysis.

1. INTRODUCTION

Since the 1970s stochastic interest rate models have played a central role in finance, both in research and in practical applications. These models were immediately implemented by practitioners to accurately price financial securities, like interest rate derivatives. Their pricing, however, was mathematically far more demanding than equity pricing, due to the fact that the whole term structure of interest rates and its development over time was required. With time the interest rate option pricing models have seen a major development. They have gradually evolved from initial spot rate models to HJM framework and later to forward rate models. Over the time it has been noticed that the drawback of one model has led to the evolution and development of other. Over the last 30 years, academics and practitioners have expended significant effort in attempt to model the behavior of interest rates. Until recently, while a wide variety of interest-rate models have been developed, very few attempts have been made to compare the empirical success of alternative specifications of interest-rate dynamics.

The first and foremost issue that catches the attention in modeling of the term structure of interest rates and the pricing of interest rate contingent claims is that of arbitrage. Pricing of different assets should imply the same market risk, i.e. the model needs to be arbitrage free. Also to derive a specific pricing result for arbitrary stochastic processes, restrictions need to be placed on the form of stochastic processes so that they give closed form solutions that is well within the bounds and is not absurd. However in the context of interest rate modelling of term structures, more than one underlying asset is involved as the prices of contingent claims depend on the current prices of bonds of all maturities along the term structure. There are two types of arbitrage conditions, viz, correct price of bond with respect to its future price and also the correct price movement of bond with respect to the prices of other bonds involved so as to avoid any arbitrage.

Another issue that arises is whether to specify the current term structure in terms of spot rates or forward rates. Although at broad conceptual level, both the specifications would be equivalent but the input data and details of model will be different depending upon the stochastic process assumed. It’s the number of factors used to describe the term structure that is the basic difference. The simplest one is the one factor so that there is a perfect correlation between the bond prices and the factor. If it’s a multifactor model then the relationship between the factors needs to be checked so that the model is arbitrage free. However it should be kept in mind that a long term bond is a bundle of short term bonds in a risk neutral world and hence the movements of various factors have to be tied together. For the sake of computational ease, the number of factors used has been limited to two and the same have been used in the short term interest rate and the long term interest rate [Brennan and Schwartz (1979)], the short term interest rate and a spread between the long-term and short term interest rates [Schaefer and Schwartz (1984)] the short term interest rate and the rate of inflation [Cox, Ingersoll and Ross (1985b)] and the short term interest rate and its volatility [Longstaff and Schwartz (1992)], the short rate and duration [Schaefer and Schwartz(1984)] and the short rate and its mean [Das and Foresi (1996)].

Also to be taken care of are the distributional assumptions. The discontinuities in the price path of underlying asset can be accommodated in the models only with the imposition of risk neutrality that correlates perfectly with the jump component. The empirical evidence in the support and existence of mean reversion is weak but the researchers feel that it is built into high frequency data models. If the criterion of analytical tractability is used to obtain closed-form solutions for bond and contingent claim prices the candidates for the interest rate processes are the following special cases: Gaussian and the square root processes (for which the elasticity parameter is one-half) and the lognormal case under some special cases [Subrahmanyam,(1996)]

The purpose of this study is to perform an empirical analysis of the various interest rate models. However I have confined myself to the boundaries of only the short rate models. It includes Vasicek, Cox Ingersoll and Ross (CIR hence forth) and Hull-White model. These models have been studied on the wide number of parameters with regard to the evolution of the theory, the improvements in the models as compared to the previous ones, term structure models, empirical tests and the method of implementation. The study is a combination of qualitative and quantitative research done on the various works performed by others. The models have been tested for their performance and robustness on the MIBOR 1-month and 3-month data. The relevance of the process followed by the short rate, and its implication for the term structure of interest rates, is that it has a direct bearing on the pricing of interest rate derivatives.

This thesis is organised as follows. Section 2 reviews the literature. Here the emphasis is given on the inception of the model, its development, the various techniques and critics by the eminent people and comments. Theory involved in the study of short-term interest rates and the development of a research framework are presented in Section 3. Section 4 however concentrates on the methodology used in the research work. Section 5 throws some light on the data used. This section introduces the not so famous MIBOR (Mumbai Inter Bank Offer Rate). It also explains about the basics of how this MIBOR is calculated and disseminated to the market. Section 6 is about the empirical study and the various results that came into light from the study. Finally section 7 concludes the research work with the crux of the findings and also defines the future scope of this work.

2. LITERATURE REVIEW

The most widely accepted practice for modelling interest rate models is the idea that they can be represented by a stochastic process. Thus the theory is probabilistic in nature and the same has been used in the works of Dothan (1990) etc. A vast research work has been performed in this area and the literature published is humongous. However is only a set of few papers that have become a benchmark in the study, the rest of the works have been considered as the extension of these papers. It is beyond the scope of this thesis to put forward all the theories and the works done. It would however be useful to have a gist of main features of these papers and then decide the purview of study to have a better command in the area and to point out the lacunae in the present paradigm.

More recently the research on the no arbitrage pricing theory stared with Black and Scholes (1973). They proposed the very famous Black Scholes equation for option pricing. They actually started a whole new theory on term structure modelling and pricing of assets. They came out with an innovative concept and the relationship amongst production technology and term premia.

Many of the interest rate models that specify the dynamic behavior of the short term riskless rate have been developed in the continuous time framework. Some of these models are by Merton (1973 a & b), Vasicek (1977), Dothan (1978), Brennan and Schwartz (1979), Cox-Ingersoll-Ross(1985), Hull and White (1990) , Black Karasinski (1991)etc.

Merton in 1973 stated that the riskless rate follows a Brownian motion with drift. He used the stochastic process and thus derived a model of discount bond prices. Vasicek in 1977 used the Ornstein – uhlenbeck process and derived an equilibrium model of discount prices. The model has been developed on the Gaussian process and the same concept has been used by Jamshidian(1989)

However the most widely accepted model for the valuing of contingent claims is undoubtedly the Cox-Ingersoll-Ross model proposed by Cox et al in 1985. This model has been widely accepted and has been used extensively in development of valuation models for contingent claims that are interest rate sensitive. Dunn and McConnell (1981) have used this method for valuation of mortgage backed securities, Cox et al (1985) have used it in the discount option model, Ramaswamy and Sundaresan (1986) have used it in the futures and futures option pricing models in their works, Longstaff (1992) has used it in yield option valuation and the list continues.

Vasicek model is the harbinger of CIR model and they share the common theoretical built up. Hence the academicians study these models together. Since Vasicek model can allow the negative interest rates it lost the interest and curiosity of researchers. One of the extensive studies of the Vasicek and CIR models was carried out by Brown and Dybvig (1986) in the early days. They estimated the one factor CIR model of term structure. They used the monthly quotes of US treasury issues from 1952 to 1983 and found that it is very much possible to estimate the implied and short term zero coupon rates and implied variance of changes in short rates. Their works looks convincing because of the following reasons. First of all the approach imposes smoothness across the term structure by using the functional form for bond prices that derives from an economic model. This depends on small number of parameters in comparison to other approaches like splines approach. Both the authors pointed that expected risk premiums are reflected in the term structure. That's because the term structure in CIR model has the information currently available to market about the future course of events.

Keim & Stambaugh (1985) suggested that ex ante yield curve information may have some predictive power for future rates of return. However Brown & Dybvig's work failed to address the relationship between the yield curve and expectation value of future interest rates. Brown & Dybvig's model could be casted explicitly in terms of notional bonds, but it failed to relate the single factor model to inflation. Also according to them, the interest rate volatility estimated from cross sections corresponds to the standard deviation obtained from time series. Hence it was quite evident at that time that CIR model is able to capture a fundamental feature of reality.

The ability of models to predict the dynamics of yield curve was questioned. The findings suggested that a great care should be taken into consideration while calculating the risk – premium and option pricing terms by using Vasicek and CIR models.

CIR model is an equilibrium model with analytical expressions for the equilibrium interest rate dynamics and bond prices. Unlike arbitrage models like Vasicek, the equilibrium models insure internal consistency and make no assumptions on the risk premium required by investors for bearing interest rate risk. CIR model is practically quite efficient and useful for pricing the options, hedging and formulating trading strategies. It also explains the effect of various economic variables on the term structure.

Brown and Dybvig (1986) utilised neither the conditional nor steady state density. This property has been studied by Gibbons and Ramaswamy (1993). They used the steady state density of the interest rate in testing the CIR model and tested the theory of the term structure of indexed bond prices. They used Hansen's GMM model using probability distribution of single state variable without using the aggregate consumption data. This technique enabled them to estimate a continuous time model based on discretely sampled data. When the theory was tested, the results came quite positive for CIR model for index bonds. The term premium part came out and the term structure could take several shapes, giving allowance for both upward and downward sloping term structure for indexed bonds.

The Gibbons and Ramaswamy (1993) model had many advantages over the previous ones. Firstly by using the continuous time approach the misspecifications arising from the temporal aggregation were avoided. Also as it used the Hansen's approach, use of aggregate consumption data was avoided thus making the measurement process simpler. Finally the model was such that no stochastic specification for aggregate price level was necessary and made the grasp, understanding and usability simpler.

Pearson and Sun (1994) proposed an empirical method using the conditional density function of state variables to estimate and test CIR term structure model. They tried to recover the unobserved state variables from observed market prices and thus utilise the conditional density function for estimation and testing. Conditional density function determines the evolution of term structure through time so they used the method of maximum likelihood to estimate the parameters of CIR model. The main advantage of their model was the use of the time series information to estimate term structure model. They performed various empirical tests on several treasury bills, notes and bonds maturing at different times. The results showed that the estimates based on bills show large pricing errors. Hence they rejected the CIR model by likelihood ratio test. Also according to them the CIR model failed to describe the bond price data and also there were large errors in estimates based on short end of yield curve.

From the above analysis it is quite clear that data on bond prices lacks accuracy. So called "off-the-run" bonds are more thinly traded than the "on-the-run" bonds. Thus for more accuracy the use of Eurodollar future prices was one of the suggested alternatives. Along with it, Kalman filter estimation techniques as described in Jegadeesh & Pennacchi (1996) tried to plug the loop holes and rectify the flaws.

Summers & Heston (1991) modified the economic approach of Gibbons and Ramaswamy (1993) that allowed the academicians to work with bond returns in excess of return on short term Treasury bill without relying on the inflation data. Commendable work was done by Brown and Schaefer (1994) by employing the cross sectional method. The model was quite simple but it failed to capture all the dynamic information in the data and also failed to identify all the parameters in the CIR model.

Till the early 90's a whole gamut of researchers tried to describe the yield curve and term structure movements using one factor models. But unfortunately none of them has been very successful. This inability of one factor models to fit the nominal value of nominal government bond has led to the development of multiple variable models. The first step in this direction was taken by Brennan & Schwartz (1982) and Nelson & Schaefer (1983) where they considered two state variable models specifying the factor risk premiums exogenously. But the major development took place later when the researchers took the current term structure in the way it's given and developed a no arbitrage yield curve model that fits perfectly with the data. Ho & Lee (1986) gave a new direction to the previous research in the no-arbitrage yield curve model. They proposed a binomial tree approach for bond pricing that fitted exactly to the current term structure of interest rates. Black, Derman and Troy [(1990), BDT henceforth] modified the Ho & Lee model and thus proposed a binomial tree approach to construct one factor model of the short rate that fits the current volatilities of all discount bond yields as well as the current term structures.

Hull and White (1990) suggested two one factor models of the short rate that are also capable of fitting both current discount bond yield volatilities and the current term structure of interest rates. According to them, it is possible to determine the process parameters and short term interest rate from the market data. They also developed what is known as the extended Vasicek Model that allows time dependent speed of reversion. However the model suffers with some problems in the long term reversion levels just like Vasicek and CIR models. Thus one stratum of academicians also says that the main purpose of a model is to provide hedging and pricing results than to describe the term structure movements. Under such statements, CIR, Vasicek and Hull White models are very reliable.

Rebonato (2000) favors Hull & white model by stating some of the positive features of the model. But it also states that the model needs to be handled carefully. The Hull & White model has been penalised and criticised for allowing the negative rates, but because of the presence of means reversion factor in the model, the probability of such an occurrence is almost negligible. The most important point of the Hull & White model is the fact that it can account even for the prices of the money caps encountered in several yield curve / cap volatility curve environments. The implementation with time varying reversion speed and volatility has been found to imply implausible behavior for the term structure of volatilities as seen from future point in time. If these shortcomings are recognised, the existence of analytic solutions for bond prices gives the Hull & White model great appeal as long as it is implemented judiciously and carefully.

Also worth mentioning is the work of Chan, Karyoli,Longstaff and Sanders (1992) in which they studied various continuous time models of short term riskless rate using the Generalized Method of Moments. They compounded the performance of various models like Merton, Vasicek, CIR, Dothan etc. Their major emphasis was on the ability of these models to capture the volatility of term structure. This property was quite important as the volatility governs the riskless rate which in turn determines the prices of contingent claims. They also found that models that allow conditional volatility of interest rate changes to be highly dependent on interest rates are the ones that describe the interest rate dynamics in the best manner. However they specifically commented that Vasicek and CIR models fail to capture the relationship between the volatility and term structure models. But in the wake of other computational and valuation problems in other models, an effort has been made to define and study the term structure using these models.

CONCLUSION

The purpose of this research work was to test whether Vasicek, Cox-Ingersoll-Ross and Hull & White one factor models are adequate and appropriate for explaining and reproducing the term structure accurately from the given MIBOR 1-month and MIBOR 3-month rates. The effort has been made to not only compare the market yield curves with the model yield curves but also a comparison has been made to compare and contrast the results from the different models as well.

One of the key findings that came into light was that in the wake of ambiguity in the choice of reversion speed, volatility and mean reversion levels along with the stability of the estimation, parameters have been the biggest empirical problems of Vasicek, HW and CIR models. Due to the same reason all the three models can give different bond prices and same yield curve can be poorly reproduced by these models because of the limited flexibility of the function that defines the discount factor. This would further result in the mispricing of the bond option.

The intuition behind the whole work has been to construct the yield curves of the MIBOR and calculate if there are any arbitrage opportunities available arising from the difference in the yields to maturity in the primary bond market and the secondary bond market. The results showed that the CIR and Vasicek models are robust models but somehow HW model failed to capture the initial term structure to perfection and there exists arbitrage opportunity with the same. However the fact remains that the models used were one factor models. This in turn defines the future scope of the research work done. Further on the same data we the can use 2 factor models and it would be interesting to see the replication of term structure by 2 factor models.

APPENDIX

A.1 APPENDIX A

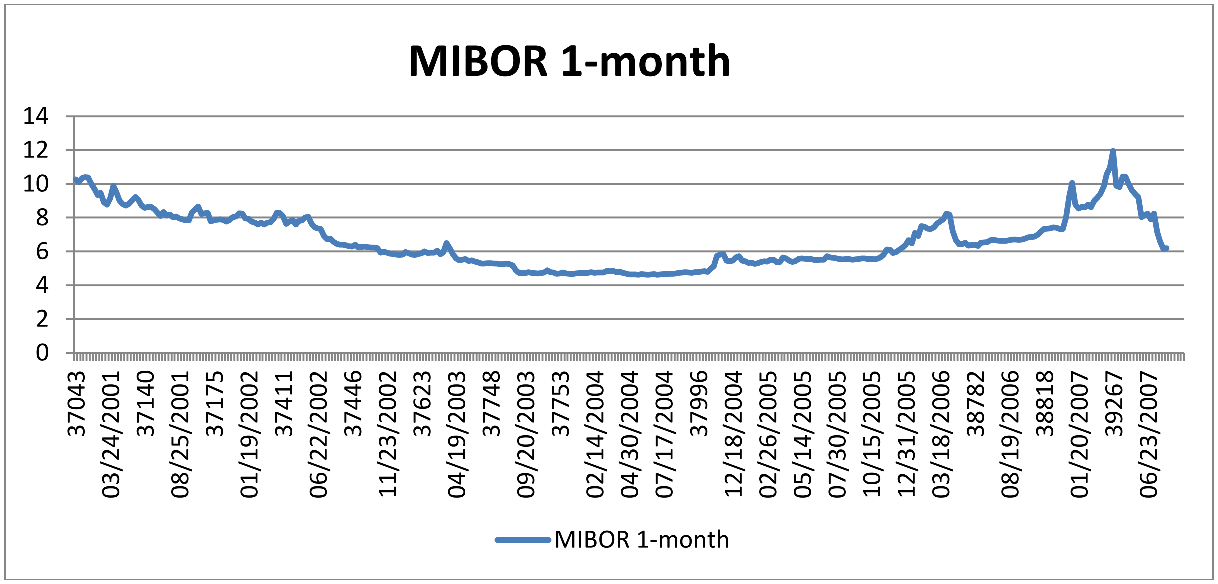


Fig19 : MIBOR 1-month curve

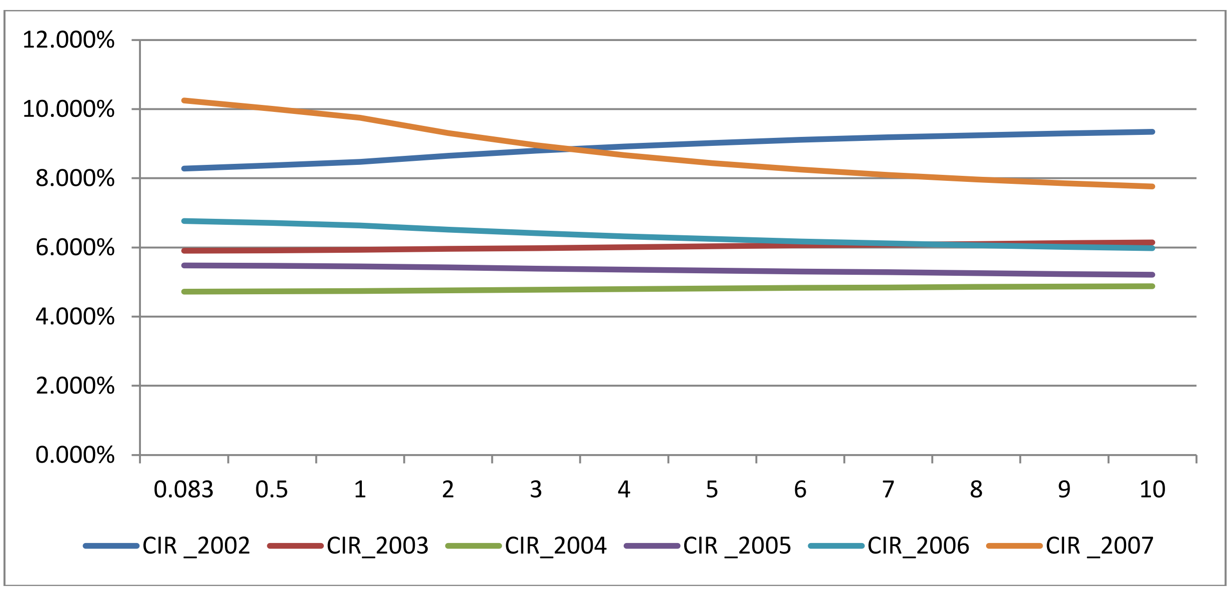


Fig 20 : Yield curve graph for the CIR model for the year 2002-2007(MIBOR 1-month)

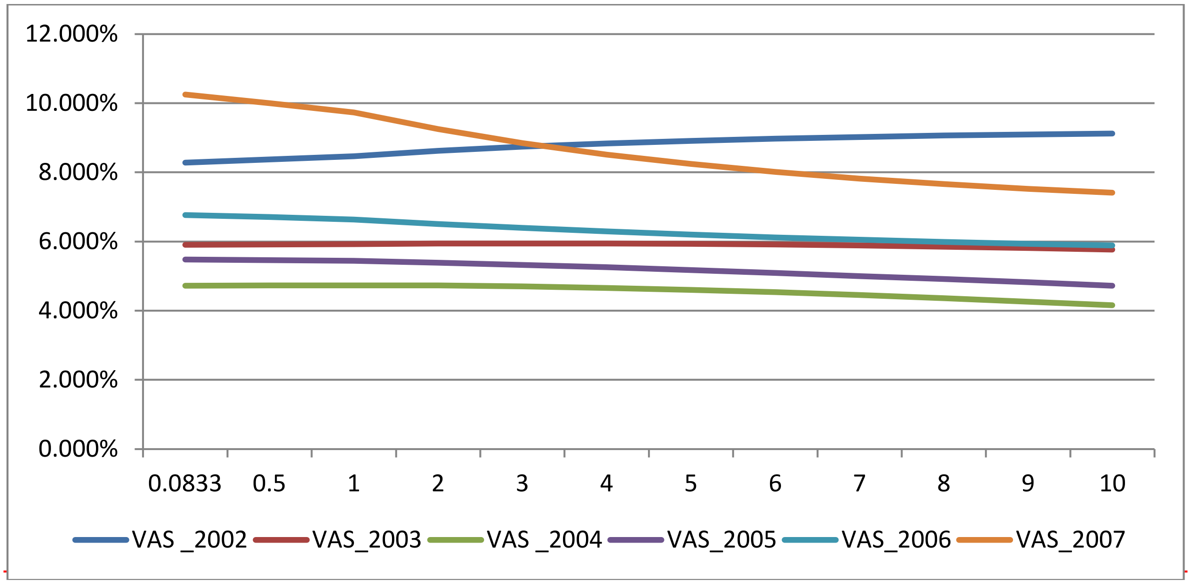
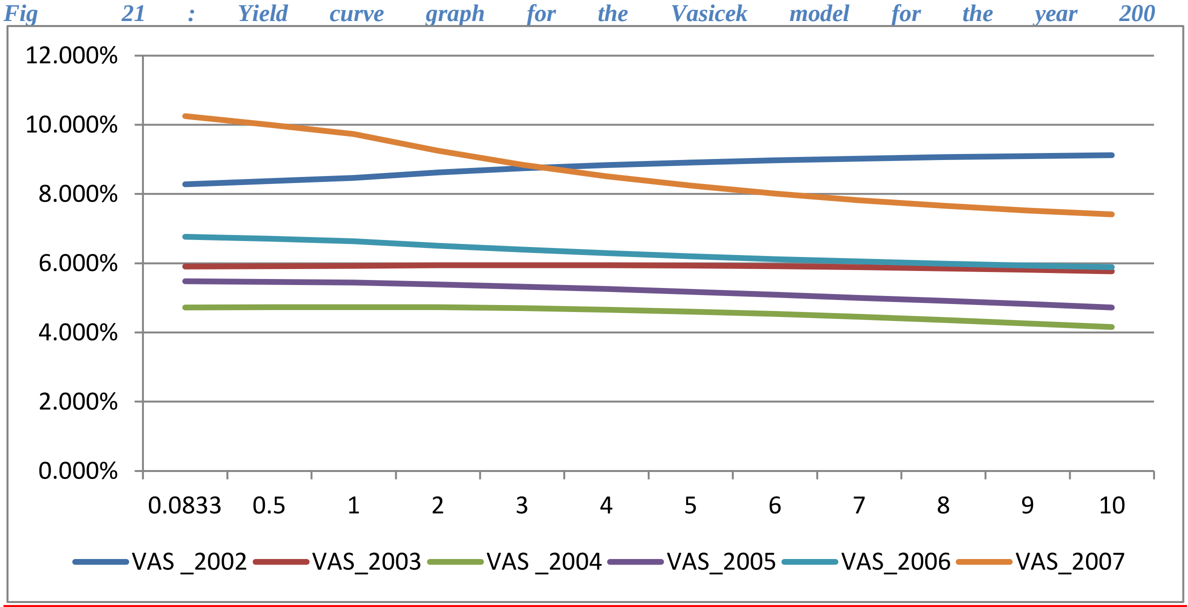


Fig 21 : Yield curve graph for the Vasicek model for the year 200



2-2007(MIBOR 1-month)

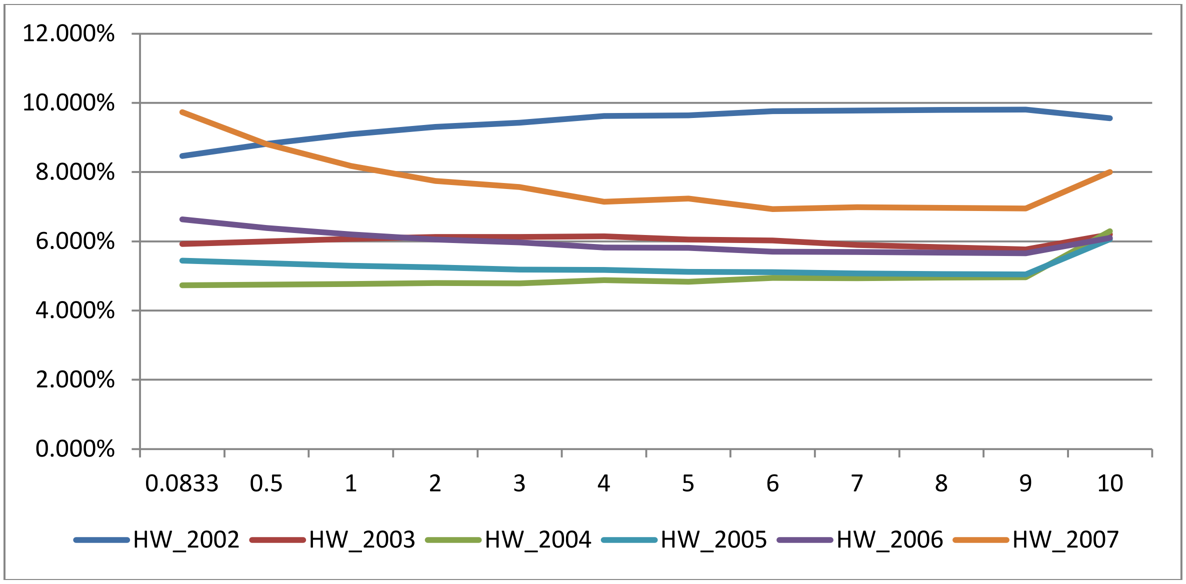


Fig 22 : Yield curve graph for the Hull white model for the year 2002-2007(MIBOR 1-month)

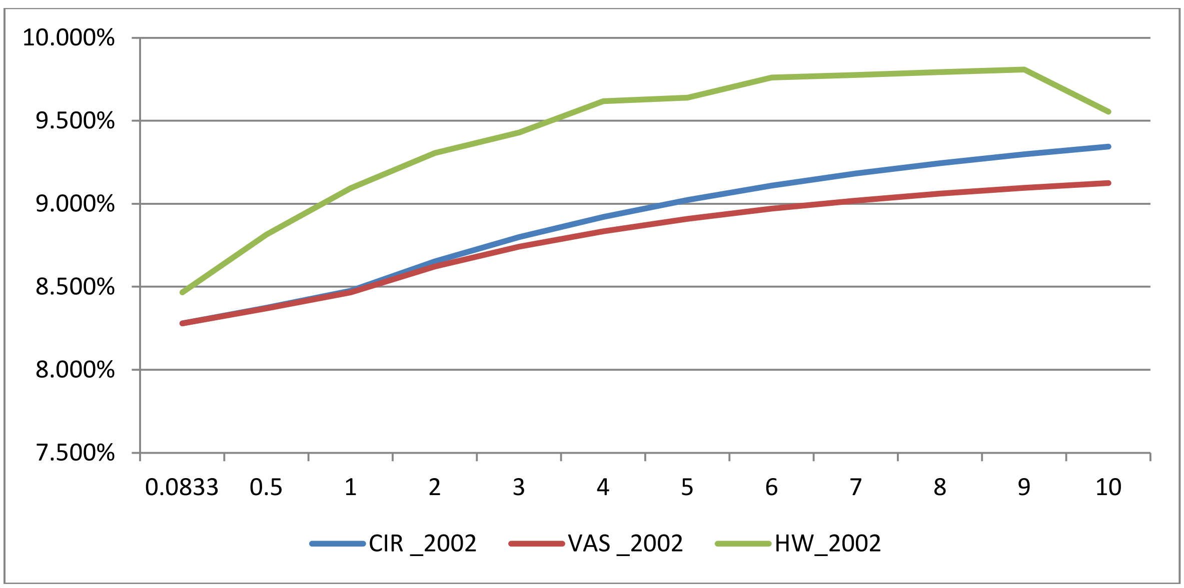


Fig 23 : Yield curve graph for the year 2002 (MIBOR 1-month)

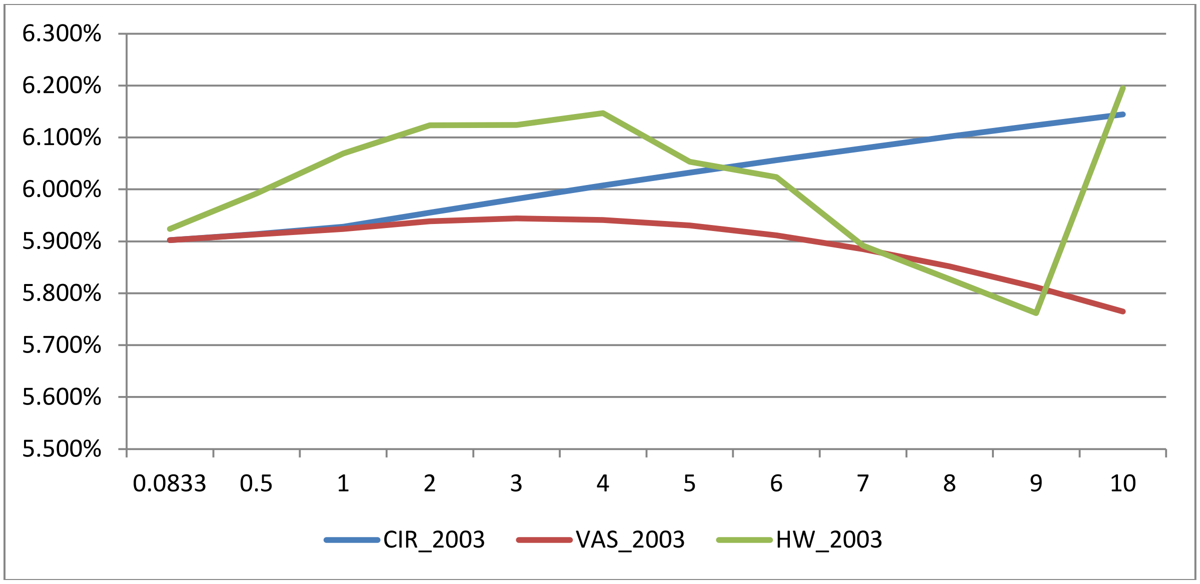


Fig 24 : Yield curve graph for the year 2003 (MIBOR 1-month)

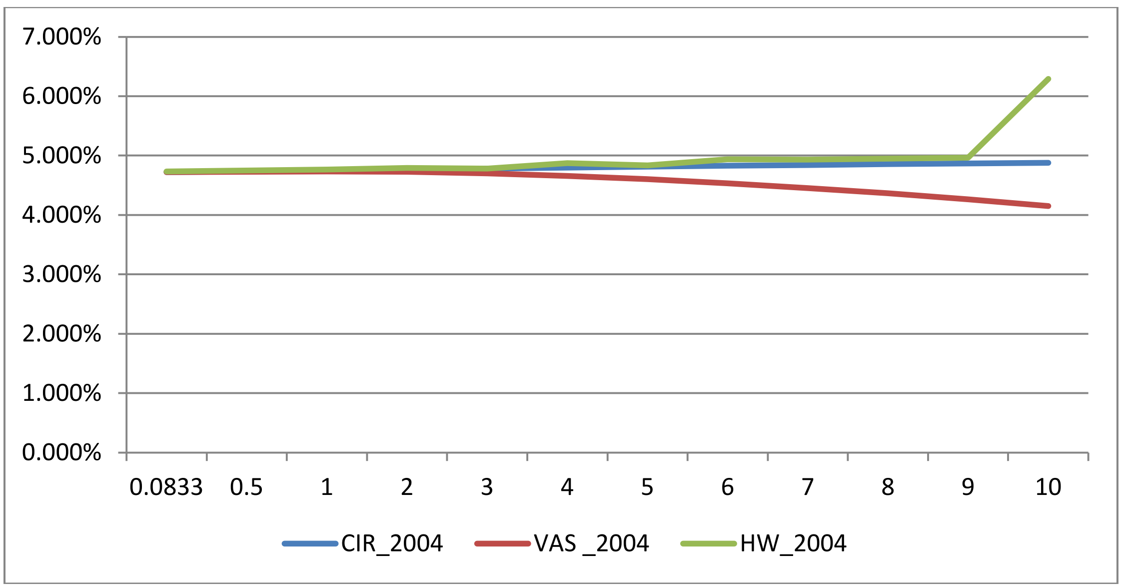


Fig 25 : Yield curve graph for the year 2004 (MIBOR 1-month)

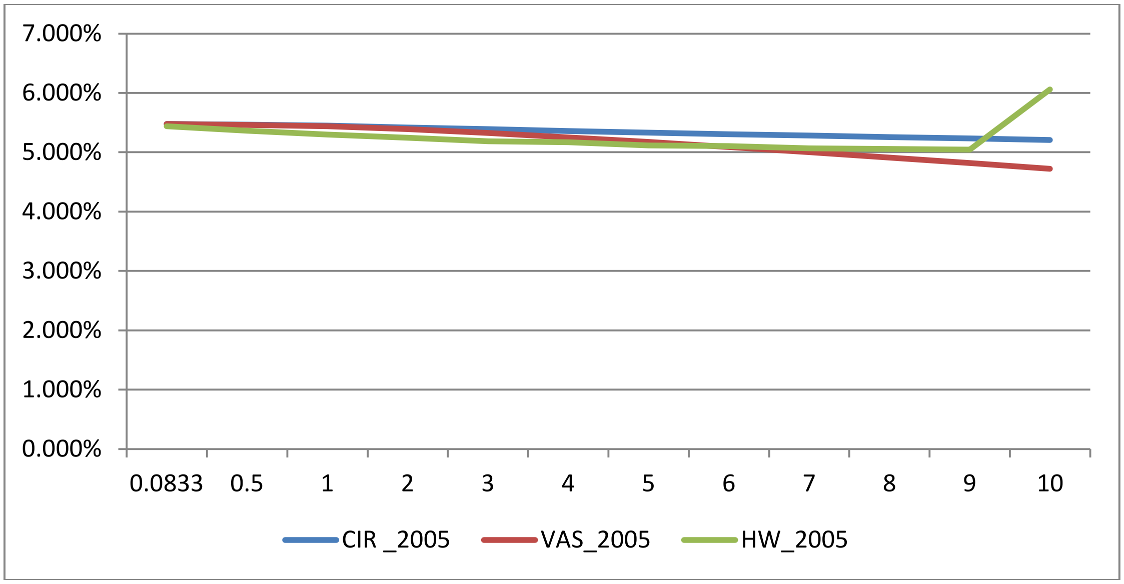


Fig 26 : Yield curve graph for the year 2005 (MIBOR 1-month)

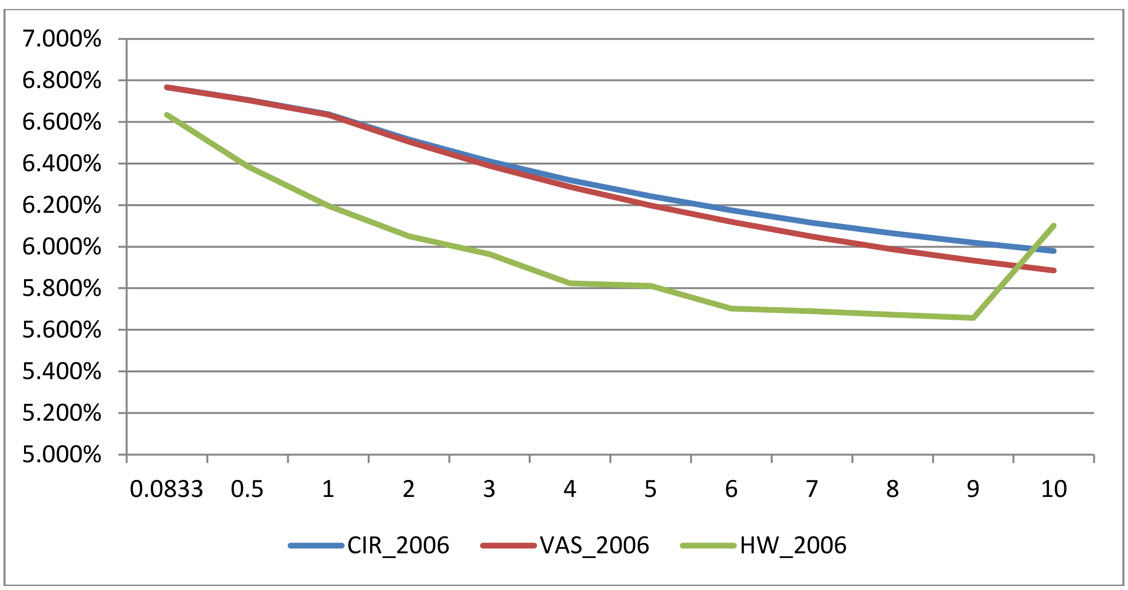


Fig 27 : Yield curve graph for the year 2006 (MIBOR 1-month)

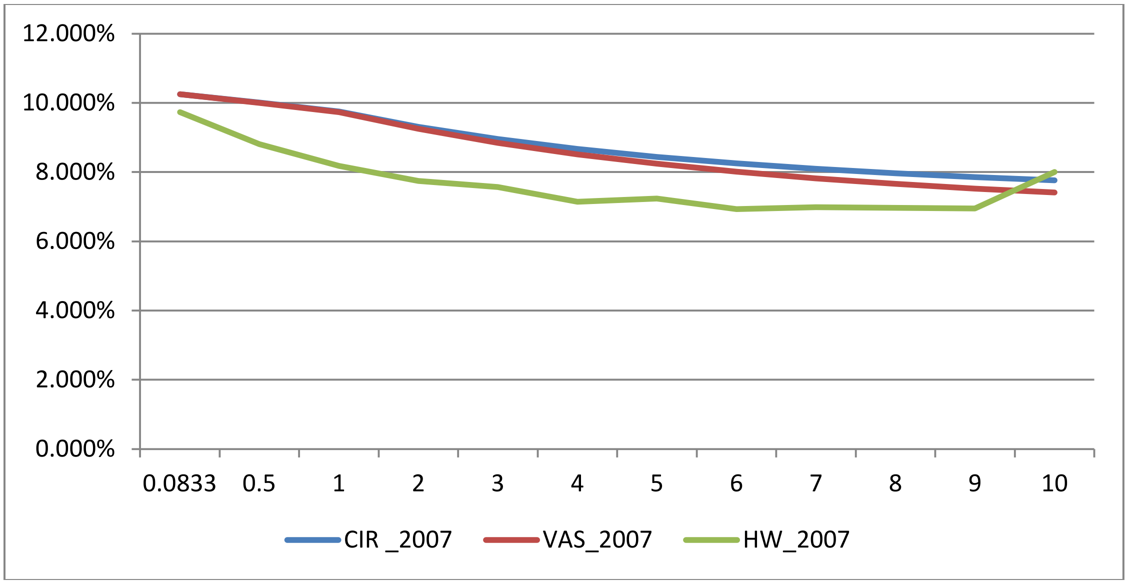


Fig 28 : Yield curve graph for the year 2007 (MIBOR 1-month)

A.2 APPENDIX B

Time to Maturity	CIR 2002	CIR 2003	CIR 2004	CIR 2005	CIR 2006	CIR 2007
0.0833	6.33804E-08	1.863E-08	7.7196E-09	1.48266E-08	3.66E-08	1.995E-07
0.5	1.10534E-06	5.601E-07	2.2767E-07	6.04537E-07	1.956E-06	1.312E-05
1	1.31727E-06	1.767E-06	7.0083E-07	2.73232E-06	1.094E-05	8.188E-05
2	4.27965E-07	4.202E-06	1.5712E-06	1.33286E-05	6.734E-05	0.0005337
3	1.63965E-05	5.163E-06	1.783E-06	3.48921E-05	0.0001954	0.0015161
4	6.65523E-05	4.382E-06	1.3356E-06	6.96484E-05	0.0004049	0.002992
5	0.000154387	2.548E-06	6.0239E-07	0.000118835	0.0006913	0.0048111
6	0.000272721	7.281E-07	6.1544E-08	0.000182726	0.0010407	0.0067906
7	0.000409372	5.534E-09	1.4882E-07	0.000260744	0.0014345	0.0087633
8	0.000551498	1.287E-06	1.1886E-06	0.000351616	0.0018529	0.010599
9	0.00068806	5.209E-06	3.3726E-06	0.000453541	0.0022773	0.0122086
10	0.000810832	1.212E-05	6.7661E-06	0.000564363	0.0026916	0.0135407
MSE	**0.000247728**	**3.166E-06**	**1.4805E-06**	**0.000171087**	**0.0008891**	**0.0051542**
MSE %	**0.02477%**	**0.00034%**	**0.00016%**	**0.01853%**	**0.09632%**	**0.55837%**

Fig 29 : Mean Square Errors for CIR Model (MIBOR 1-month)

Time to Maturity	VAS 2002	VAS 2003	VAS 2004	VAS 2005	VAS 2006	VAS 2007
0.0833	6.384E-08	1.87715E-08	7.784E-09	1.495E-08	3.69E-08	2.012E-07
0.5	1.13E-06	5.67569E-07	2.384E-07	6.195E-07	1.968E-06	1.33E-05
1	1.512E-06	1.87097E-06	8.517E-07	2.977E-06	1.113E-05	8.481E-05
2	2.683E-08	5.45547E-06	3.568E-06	1.738E-05	7.045E-05	0.0005776
3	7.824E-06	1.00462E-05	1.104E-05	5.584E-05	0.0002098	0.0017027
4	3.472E-05	1.65748E-05	3.055E-05	0.0001364	0.0004449	0.0034644
5	8.135E-05	2.69275E-05	7.575E-05	0.0002807	0.0007752	0.0057139
6	0.0001427	4.39359E-05	0.0001685	0.0005123	0.0011877	0.0082398
7	0.0002119	7.15547E-05	0.0003398	0.0008539	0.0016623	0.0108322
8	0.0002824	0.00011499	0.0006295	0.0013259	0.002176	0.0133158
9	0.0003489	0.000180699	0.0010862	0.0019448	0.0027059	0.015562
10	0.0004078	0.000276268	0.0017657	0.0027225	0.0032316	0.0174882
MSE	**0.0001267**	**6.2409E-05**	**0.0003426**	**0.0006544**	**0.0010398**	**0.0064162**
MSE %	**0.01382%**	**0.00676%**	**0.03712%**	**0.07090%**	**0.11264%**	**0.69509%**

Fig 30 : Mean Square Errors for Vasicek Model (MIBOR 1-month)

Time to Maturity	HW 2002	HW 2003	HW 2004	HW 2005	HW 2006	HW 2007
0.0833	1.22E-08	1.44E-08	6.44E-09	2.28E-08	8.72E-08	7.04E-07
0.5	8.16E-07	1.54E-07	1.59E-07	1.56E-06	8.14E-06	7.86E-05
1	1.66E-05	5.41E-09	3.6E-07	9.1E-06	5.19E-05	0.000498
2	0.000116	5.91E-07	5.64E-07	4.43E-05	0.000255	0.002269
3	0.000303	1.21E-06	1.62E-06	0.000119	0.000601	0.004801
4	0.000679	4.28E-06	1.77E-06	0.000202	0.001219	0.00934
5	0.000927	7.38E-07	2.58E-12	0.00036	0.001718	0.011616
6	0.00139	4.62E-06	2.08E-05	0.000485	0.002623	0.016872
7	0.001629	6.67E-05	2.24E-05	0.000679	0.003219	0.018823
8	0.001833	0.000142	3.58E-05	0.000833	0.003814	0.020998
9	0.001997	0.000256	5.32E-05	0.00099	0.004379	0.022711
10	0.001318	3.71E-05	0.006495	0.000516	0.002064	0.011284
MSE	**0.000851**	**4.28E-05**	**0.000553**	**0.000353**	**0.001663**	**0.009941**
MSE %	**0.09281%**	**0.00464%**	**0.05987%**	**0.03828%**	**0.18012%**	**1.07693%**

Fig 31 : Mean Square Errors for Hull White Model (MIBOR 1-month)

	2002	2003	2004	2005	2006	2007
CIR	0.005401%	0.000522%	0.000242%	0.016264%	0.054174%	0.211769%
Vasicek	0.017391%	0.234278%	0.016272%	0.048017%	0.079439%	0.294912%
HW	0.013228%	0.795425%	0.022831%	0.031453%	0.112198%	0.437683%

Fig 32 : Comparison of Mean Square Errors (MIBOR 1-month)

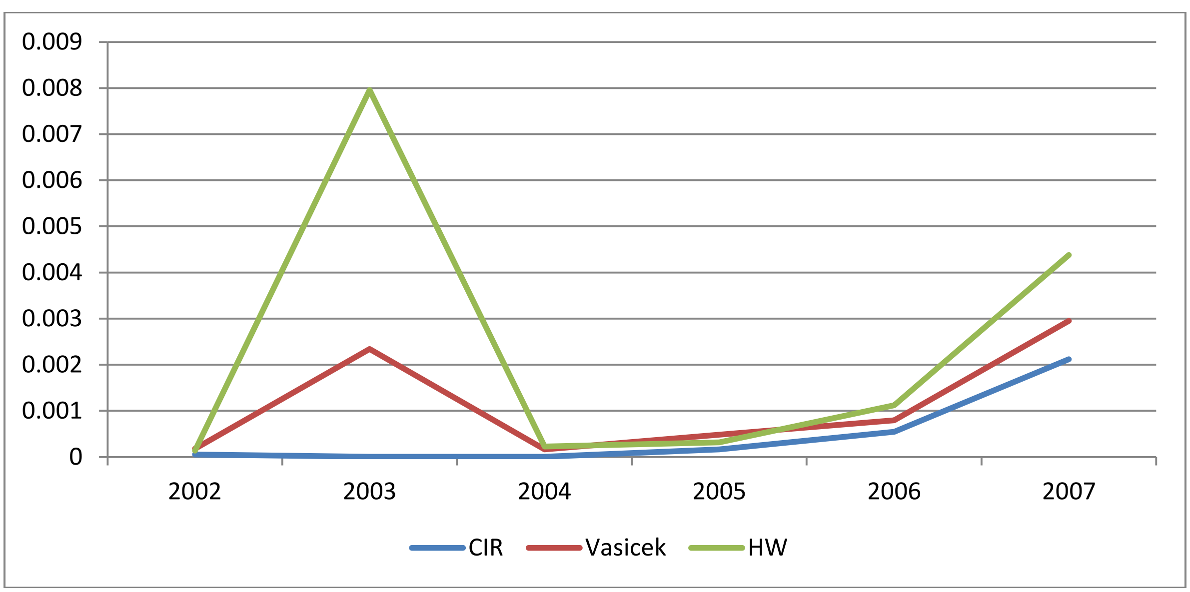


Fig 29 : Graph of Mean Square Errors (MIBOR 1-month)